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On the Non-intrusive Determination of Electron Density in the Sheath of a Spherical Probe

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14. ABSTRACT
A new method of determining probe/antenna plasma sheath parameters has been developed. For low neutral pressure environments in the absence of a magnetic field, the method provides these quantities from a probe/antenna surface through the sheath, pre-sheath, and into the bulk plasma. For high pressure environments, where conventional resonances are strongly damped, it can determine bulk plasma density. The paper cites commonly used methods of finding electron density/temperature and points out the inadequacy of each to make sheath measurements in the general case. The primary advantage of the method lies in the fact that a non-perturbative rf signal is applied to the probe/antenna for the determination and this makes the measurement independent of complicating influences such as secondary electron emission, surface conditions, and ion mass. Because of this, the ac impedance depends on frequency but not on the dc voltage unlike a typical Langmuir probe. In this paper we concentrate on the low pressure regime.

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On the non-intrusive determination of electron density in the sheath of a spherical probe

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I. Abstract

Over the last year the Charged Particle Physics Branch (Code 6750) has developed a technique which differs significantly from conventional techniques used to determine plasma density and temperature. For low neutral pressure environments in the absence of a magnetic field, the method provides these quantities from a probe/antenna surface through the sheath, pre-sheath, and into the bulk plasma. For high pressure environments, where conventional resonances are strongly damped, it can determine bulk plasma density. The technique can be employed with a planar, cylindrical or spherical probe/antenna geometry using a low level, non-perturbative rf signal. Both low and high neutral pressure regimes are characterized by the two experimentally determined components of the ac impedance, i.e., Z = Re(Z) + iIm(Z) where Re(Z) is the resistance and Im(Z) is the reactance. Both are used as plasma diagnostics. In the "collisionless" low pressure regime, where both the resistance and reactance diverge strongly at the local plasma frequency, the electron density and temperature from the probe surface to the bulk plasma can also be determined. At high pressures where $\nu >> \omega_p$ the resonance is strongly damped, but the resistance is controlled by the ratio of the bulk density to ν in certain frequency regimes. In this case measuring Re(Z) allows one to find the bulk plasma density since ν is typically known to within a factor of two or so. These are measurements which are difficult to make without the method we use since the small applied rf signal has little effect on the sheath. Because of this, the ac impedance depends on frequency but not the dc voltage unlike a typical Langmuir probe measurement. Secondary electron emission and surface conditions also have little effect, and the massive ions have almost no effect either unless the plasma is highly electronegative. In the paper here we concentrate on the low pressure regime.

II. Introduction

The determination of electron sheath density (or potential profiles) associated with either instrument or wall interface to plasma is a subject area that has been studied both theoretically and experimentally extensively over the years. The field traces its origins to the very early seminal works in the study of plasma. From interpretation of probe data in space plasma measurements to plasma confinement in fusion devices, the determination of charged particle density profiles near boundaries is crucial to interpretation of experimental results and/or diagnostic measurements. At the same time, methods used to determine electron density and temperature, as made in the bulk plasma, are often not possible near plasma wall interfaces due to the size of these regions compared

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to either the size of the instrumentation or its measurement technique. By determining sheath sizes and structure in the main plasma, inferences can be drawn with respect to these values through the plasma where boundaries exist between probes/antennas/walls.

The most widely used technique still today for determining local plasma electron density is to measure the dc impedance of a small probe placed in the plasma. Langmuir probes rely on measurement of the dc impedance which develops across an electron depleted sheath whose density structure is often estimated with the use of various models. The use of these probes is based on the ability to provide relatively precise spatial resolution and the fact that one can infer the electron energy distribution from the current-voltage curve and hence electron density and temperature. As these probes in almost all applications measure dc impedance, assumed sheath and pre-sheath models are necessary in order to interpret the measurement. This is so because the dc potential applied to the probe affects the sheath strongly and therefore the determination of the dc impedance. Because of this, impedance models of the sheath are used to relate the bulk electron density to the dc impedance in different plasma regimes. There are various complicating factors to these analyses including secondary electron emission, ion collisions, contaminants on the probe surface, non-Maxwellian velocity distributions, negative ions, and plasma production and decay within the sheath. Most of the work on Langmuir probes has been developed using weakly collisional plasma approximations where the ion flow velocity is well known from the Bohm condition. When plasmas become highly collisional, the analysis is further complicated because ion flow velocity at the sheath edge is no longer well known and the electric field extends beyond the sheath. The absence of a well-defined boundary condition considerably complicates the solution of the Poisson equation and makes modeling, or assumptions as to pre-sheath size, essential if this solution method is attempted.

Other *in-situ* techniques found both in the laboratory and in space include emissive probes and rf resonance probes. In addition there are non-intrusive techniques using microwaves or Laser Induced Fluorescence (LIF) which work well in some applications. We discuss briefly each of these methods below. In none of the analyses or experimental work has it been possible to construct an electron density profile from the probe surface, through the sheath and pre-sheath, into the bulk plasma. The analysis and experimental measurements we have developed allow such a construction.

The use of direct *in situ* probes, whether Langmuir probes, emissive probes, or conventional rf probes is based on the need for good spatial resolution and the ability to infer the electron density from the measurement. Conventional Langmuir probes[1] collect current using a voltage sweep and from these data the probe characteristic is constructed. From the characteristic, temperature and density are determined using standard theory[2]. In the case of emissive probes, accurate measurements of plasma potential can be obtained by operating the probe just at the point of zero emission[3]. Emissive probes can function also as Langmuir probes below the emission voltage level. Finally, rf impedance probes are widely used to determine the bulk electron density both in the laboratory and in space[4,5]. As mentioned above, since all of these probes are contained in the plasma, perturbations to the plasma itself can occur for a number of reasons. From simple location perturbations such as plasma depletion to the electronics employed in Langmuir probe measurements[6], any of these devices can compromise not only the local measurement integrity, but

the entire global plasma, particularly in an rf environment. Also, in high pressure discharges, the use of Langmuir probes can be compromised when ionization lengths and mean free paths are smaller than probe size. In fact, in comparison to microwave interferometry measurements in rf discharges, Langmuir probes have been shown to give average charge densities 2 to 4 times lower (when analyzed using standard theory) at argon neutral pressures in the 250 to 500 mTorr range[7]. The method we propose is largely insensitive to pressure variations as described and thus is not subject to these drawbacks. The fact that these results can be used existing probe/object/wall measurements to determine the sheath can be extremely useful in the analysis of collected probe data.

The two techniques for measuring plasma parameters non-intrusively are Microwave Interferometry (MWI)[7] and Laser-Induced-Fluorescence (LIF)[8] coupled with Laser-Collision Induced Fluorescence (LCIF)[9,10]. As these measurements are remote from the region whose characteristics are desired, they would seem an ideal solution to the problems of *in-situ* probe perturbation of an existing environment. However, the primary drawback to these techniques is that they do not allow sufficient spatial resolution but must resort to averaging in the final analysis. In the MWI method a microwave beam (typical frequency 35 Ghz) propagates across the plasma region whose density is to be determined. The resulting phase shift between this beam and a reference signal is used to unfold the line-of-sight integrated electron density. In addition the technique must rely on a number of sources of differing frequency. This is necessary to unfold phase changes caused by the real part of the refractive index. In practice this technique relies on a presumed density profile to determine the density at the maximum of the distribution[10]. Although this is a very fast method of determining density and can be used in a variety of pulsed plasma operations, its spatial resolution is inherently poor.

Another non-intrusive measurement technique, LCIF, differs from conventional LIF in that atoms, which have already been excited to higher energy levels by the absorption of laser light, will make transitions to even higher lying energy states if they then are further excited by collisions with plasma electrons. Hence, when these atoms radiate, the radiation frequency is different. This technique is useful, however, only if electrons excite the intermediate state before that state decays. Moreover, n_e and T_e can be determined from the ratio of emitted radiation at different frequencies only if both the cross sections and the electron energy distributions are known. Unfortunately this information is rarely available. This is a serious drawback even if spatial resolution and sufficient numbers of collisions were not an issue[7]. In low pressure plasmas which are effectively collisionless, even outside sheath and presheath in the bulk plasma, the technique does not work well.

None of these methods except one[1] has been used in sheath or presheath regions. As sheaths are typically on the order of a few Debye lengths, previous methods are unable to infer a sheath/pre-sheath density profile either because of insufficient resolution, or because of the perturbative nature of the measuring probe itself.

III. Experimental Description

The primary measurement technique is the determination of the ac impedance of a probe in a plasma. A schematic representation of the experimental arrangement is provided in Figure (1) where a Network Analyzer, or a spectrum analyzer, is employed enabling a measurement of the complex impedance. We list also typical parameter values used for the low pressure investigation for which data are presented in this paper. Unlike the dc impedance, the ac impedance does not depend upon the small applied ac voltage, the presence of ions, the surface condition of the probe, secondary electron emission, or the velocity distribution. We consider dc potentials applied to the probe which vary from floating potential to larger negative biases so that for the analyses we are in the ion current collection regime. Since the rf signal applied is on the order of millivolts and therefore much smaller than any dc bias, the rf fields do not perturb the plasma, and hence the results are valid for the sheath to be expected for the biased probe. This method has broad application for space and laboratory determination of sheath structure in cases where the magnetic field is mostly ignorable.

Figure (2) is a schematic of an actual probe as employed in these measurements. The small aluminum sphere seen is connected to the network analyzer (HP8735D) through 50 Ohm coaxial cable. The cabling is 1/4 in. semirigid coax whose outer jacket and dielectric are removed at the short section near the tip to allow the center conductor to be inserted into the sphere for mounting. This arrangement is described in further detail in earlier papers.[11,12]

Historically the low pressure regime is characterized by collision frequencies which are at least two orders of magnitude below the bulk plasma frequency, and it is therefore studied in the context of weakly collisional, or *collisionless* plasma. Although this regime has been studied to a greater extent experimentally than the high pressure case, both are characterized by the two measured components of the ac impedance, i.e., $Z = \mathrm{Re}(Z) + i\mathrm{Im}(Z)$ where Re(Z), the resistance, can be thought of as a "local" diagnostic and Im(Z) as a "global" diagnostic. The commonly accepted circuit analogy to the sphere coupling to the plasma through the sheath is shown in Figure (3)[1] where it can be seen that the impedance in general has real and imaginary parts and further that the resonances can arise due to both parallel and series impedance. The sheath capacitance as well as the capacitance to the outer chamber boundaries are seen in this figure. For a more complete discussion of the basic circuit analogy see References 11,12 or Appendices I through III of Reference 13.

The context within which the term *collisionless* arises in the low pressure case is that there is a measured energy absorption which appears to maximize at approximately half the bulk plasma frequency; therefore energy is unexpectedly dissipated in a regime not conventionally characterized as collisional. By sweeping all frequencies with the Network Analyzer and measuring Z, we are able to determine bulk plasma density (by a strong resonance at the bulk plasma frequency) and the electron density and temperature from the probe surface out to the bulk plasma. Using the Boltzmann relationship, the plasma potential, the electron temperature, and the electrostatic potential, $\phi(r)$ can be determined.

As described above, the applied ac voltage has little effect on the sheath and so the ac

impedance depends on frequency but not the ac voltage. Secondary electron emission and surface conditions also have little effect, and the massive ions have almost no effect either unless the plasma is highly electronegative. Relating the bulk plasma density to the rf impedance is therefore usually easier with rf probes, especially in weakly collisional plasmas where the electron neutral collision frequency is small compared with the unperturbed plasma frequency. In such plasmas both the resistance and reactance diverge strongly at the bulk plasma frequency which is the basis of the rf probe measurement of plasma density. In highly collisional plasma the resonance is strongly damped, but the resistance is controlled by the ratio of the bulk density to the collision frequency in certain frequency regimes. The collision frequency is typically known to within a factor of two or so, and thus the bulk density can be determined to that level of accuracy.

IV. Theory and comparison to experimental results

The basic theory we have developed[13,14] is consistent with earlier works but is unique in that we are able to obtain an electron density profile from the surface of the probe to the bulk plasma in the low pressure case. In the high pressure case the method provides a new and useful measure of bulk electron density. We treat for illustration purposes the spherical probe. The primary equations for impedance which arise as a result of the application of the ac signal to the spherical probe are given by:[14]

$$Z=\frac{V_0}{I_0}=\frac{1}{I_0}\int_{r_s}^{\infty} dr\ E(r,t) \ = \ \frac{1}{4\pi\epsilon_0}\int_{r_s}^{\infty}\frac{dr}{r^2}\frac{\nu+i\omega}{(\omega_{pe}^2-\omega^2)+i\omega\nu} \tag{1}$$

from which resistance and reactance follow as,

$$R_\Omega(\omega) \ = \ \frac{1}{4\pi\epsilon_0}\int_{r_s}^{\infty}\frac{dr}{r^2}\frac{\nu\omega_p{}^2}{(\omega_{pe}^2-\omega^2)^2+\omega^2\nu^2} \tag{2}$$

and,

$$Z_r(\omega) \ = \ \frac{\omega}{4\pi\epsilon_0}\int_{r_s}^{\infty}\frac{dr}{r^2}\frac{\omega_p^2-\omega^2-\nu^2}{(\omega_{pe}^2-\omega^2)^2+\omega^2\nu^2} \tag{3}$$

where the complex impedance, Z, is given by Ohm's Law, with V_0 the applied rf voltage and I_0 the rf current driven by V_0 into the sphere and out through the plasma. Also in the above equations, r_s is the sphere radius, ν is electron-neutral collision frequency, ω is the applied frequency and ω_{pe}(r)) is the local plasma frequency.

IV.a Low pressure

In this regime collisions are negligible but there is nevertheless a “resistance” which arises and which is often termed “collisionless”. The resistance can be found by solving Eqn (2) using contour integration to treat the singularity as $\omega \rightarrow \omega_{pe}$ when $\nu \rightarrow 0$. The integration results in a principal value, which is the reactance, and the residue, which is the resistance. The resistive part is given by,

$$R_\Omega(\omega) \;=\; Re(Z_{rf}) = \frac{1}{8\epsilon_0 r_r^{\,2}}\left[\frac{d\omega_p}{dr}\right]^{-1}\Big|_{\omega_p=\omega} \tag{4}$$

It is from this expression that we are able to find the electron density profile by inversion, i.e., Eqn (4) may be integrated and re-expressed in the form,

$$r(\omega_p) \;=\; \frac{r_s}{1 - 8\epsilon_0 r_s \int_{\omega_p(r_s)}^{\omega_p} d\omega R_\Omega(\omega)} \tag{5}$$

where the integral is done numerically using the $R_\Omega(\omega)$ found in the experiment and $\omega_p(r_s)$ is the minimum frequency for which $R_\Omega > 0$. If this expression is inverted $\omega_p(r)$ and thus $n_e(r)$ may be found. Examples of this is shown are in Figures (4) through (6) where we plot results from two theoretical models versus the experimental results found by using Eqn (5). Figures (4) and (5) show density profiles for two different densities from the probe surface to the sheath edge using theory developed in our earlier work[14]. Figure (6) compares the experimental results to another commonly accepted theoretical profile which is derived without specifically requiring the existence of a presheath.[15] As can be seen from these figures the reconstruction of the density profiles agrees quite well with theoretical results based on two separate commonly accepted theoretical models.

IV.b High Pressure

At high pressure where $\nu >> \omega_p$ the resistance of Eqn(2) now becomes,

$$R_\Omega(\omega) \;=\; \frac{\omega_{p0}^{\;2}}{4\pi\epsilon_0 \nu_e \omega^2}\int_{r_s}^{\infty}\frac{dr}{r^2}\frac{n_e(r)}{n_{e0}} \tag{6}$$

and the bulk plasma controls the resistance. Assuming a thin sheath presheath, we are able to write,

$$R_\Omega(\omega) = \frac{1}{4\pi\epsilon_0 r_s \omega^2}\left[\frac{\omega_{p0}^2}{\nu_e}\right] \tag{7}$$

In this instance we are able to determine the bulk plasma frequency, $\omega_{p0,}$ and thus the bulk plasma density, $n_{e0} = m\epsilon_0\, \omega_{p0}^2/e^2$, where e and m are the electron charge and mass, respectively. We treat this regime in a future work.

In sum the method based on the measurement of the rf impedance allows us to measure plasma quantities in a unique fashion and in situations where size or other restrictions make measurement nearly impossible.

V Conclusions

We have based the determination of the electron density profile in a low neutral pressure plasma from the surface of a spherical probe to the bulk plasma on the results of our recent earlier work on collisionless resistance. By inverting a derived expression for the resistance, or Re(Z), which results from the evaluation of a contour integral, we are able to construct a profile of electron density based on the values of the resistance which is determined experimentally. We have compared these results to two commonly accepted models of the sheath, one of which[15] predicts the profile from the surface of the probe to the bulk plasma. We are currently collecting additional data sets for the low pressure case described here and beginning investigation of the high pressure regime.

Acknowledgements: This work was supported by ONR

VI. Figure Captions

Figure (1) - A schematic representation of the impedance probe in the large vacuum chamber at the Naval Research Laboratory showing the common plasma parameter regime used for the low pressure measurements presented in this paper.

Figure (2) - A schematic representation of the impedance probe itself showing sizes and cabling for mounting the probe (aluminum sphere) to the support cabling.

Figure (3) - The commonly accepted circuit model of probe coupling through the sheath to the plasma. The impedance is seen to have real and imaginary parts where the imaginary contribution in the low pressure case represents oscillatory behavior and the real part a resistive loss.

Figure (4) - A plot of experimental and theoretical[3] normalized electron density from the sphere surface to the sheath edge for an electron density of $n_{e0} = 5.91 \times 10^7 \, cm^{-3}$ and an electron temperature of $T_e = 0.68$ eV with neutral pressure near 0.3 mTorr.

Figure (5) - A plot of experimental and theoretical[3] normalized electron density from the sphere surface to the sheath edge for an electron density of $n_{e0} = 2.3 \times 10^7 \, cm^{-3}$ and an electron temperature of $T_e = 0.68$ eV with neutral pressure near 0.3 mTorr.

Figure (6) - A plot of experimental and theoretical[4] normalized electron density from the sphere surface to the bulk plasma for a bulk electron density of $n_{e0} = 5.91 \times 10^7 \, cm^{-3}$ and an electron temperature of $T_e = 0.68$ eV with neutral pressure near 0.3 mTorr.

VII. References

[1]N. Hershkowitz, in *Plasma Diagnostics,* O. Aucciello and D. Flamm, Ed. (Academic, New York, Vol 1, 1989)

[2]J.G. Laframboise, *Tech Report 100*, University of Toronto Institute for Aerospace Study (1966)

[3]L. Okuz and N. Hershkowitz, *Phys. Rev. Lett., 89*, 145001-1 (2002)

[4]A. Mishra, C.U.S. Patnaik, and P. Misra, *J. Phys. Soc, Japan, 70*, 1966 (2001)

[5]M.D. Jensen and K.D. Baker, *J. Spacecraft and Rockets*, *29*, 91 (1992)

[6]J.C. Holmes and E.P Szuszczewicz, *Rev. Sci. Instrum., 46*, 592 (1975)

[7]M.A. Heald and C.B. Wharton, *Plasma Diagnostics with Microwaves*, (Wiley, New York, 1965)

[8]R.A. Stern and J.A. Johnson, III, *Phys. Review Lett.*, *34(25)*, 1548 (1975)

[9]K. Tsuchida, S. Miyake, K. Kadota, and J. Fujita, *Plasma Phys. 25*, 991 (1983)

[10]K. Dzierzega, K. Musiol, E.C. Berick, and J. R. Roberts, *J. Appl. Phys. 80(6)*, 3196 (1996)

[11]D.D. Blackwell, D.N. Walker, and W.E. Amatucci, *Rev. Sci. Instrum., 76*, 023503 (2005)

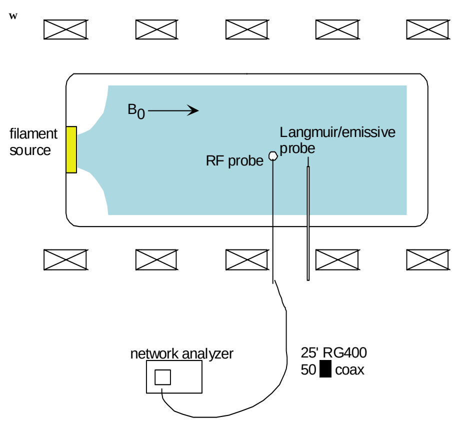
[12]D.D. Blackwell, D.N. Walker, S.J. Messer, and W.E. Amatucci, *Phys. Plasmas, 12,* 093510 (2005)

[13]D.N.Walker, R.F. Fernsler, D.D. Blackwell, W.E. Amatucci, S.J. Messer, *NRL Memorandum Report, 6750-05-8902,* pgs 16-22(2005)

[14]D.N.Walker, R.F. Fernsler, D.D. Blackwell, W.E. Amatucci, S.J. Messer, *Phys of Plasmas*, *13*, 032108 (2006)

[15]J.E. Allen, R.L.F. Boyd, and P. Reynolds, *Proc. Phys Soc. B, 70,* 297 (1957)

VIII. Figures



Magnetic Field Strength (G)	$\leq$ 5 G
Plasma Density (cm^{-3})	10^7 cm^{-3} - 10^9 cm^{-3}
Electron Temperature (eV)	0.5 eV - 1 eV
Ion Temperature (eV)	~ 0.05 eV
Debye Length (cm)	~ 0.1 cm
Neutral Density (cm^{-3})	10^{11} cm^{-3} - 10^{13} cm^{-3}

Figure 1

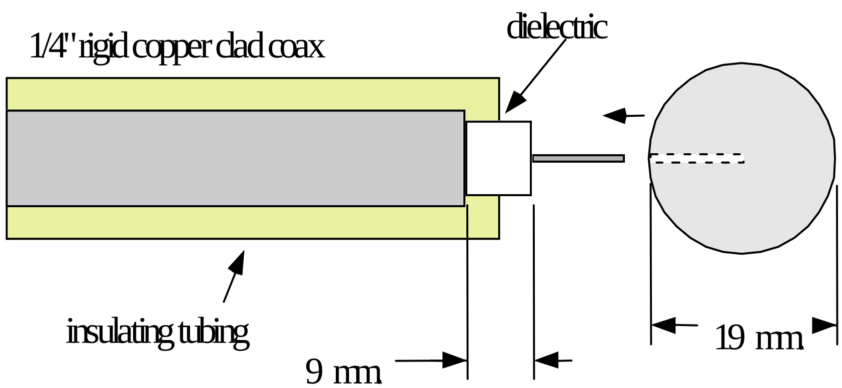
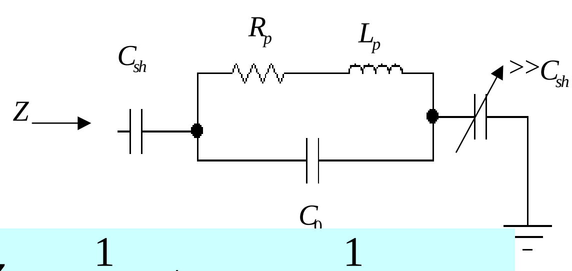
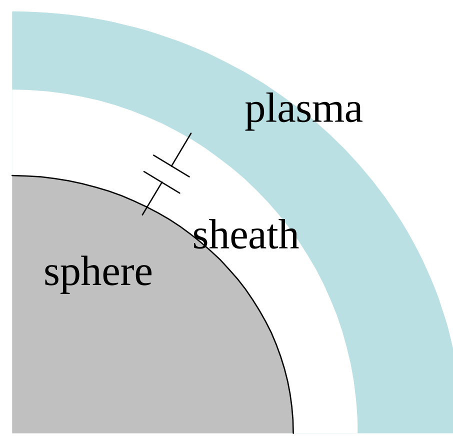

1/4" rigid copper clad coax
dielectric
insulating tubing
9 mm
19 mm

Figure 2



$$Z = \frac{1}{j\omega C_{sh}} + \frac{1}{j\omega C_0 + \dfrac{1}{R_p + j\omega L_p}}$$



$C_0 = 4\pi\varepsilon_0 R$ vacuum capacitance

$L_p = \dfrac{1}{\omega_p^2 C_0}$ inductance from electron motion

$R_p = \nu_{en} L_p$ energy dissipation from collisi

Figure 3

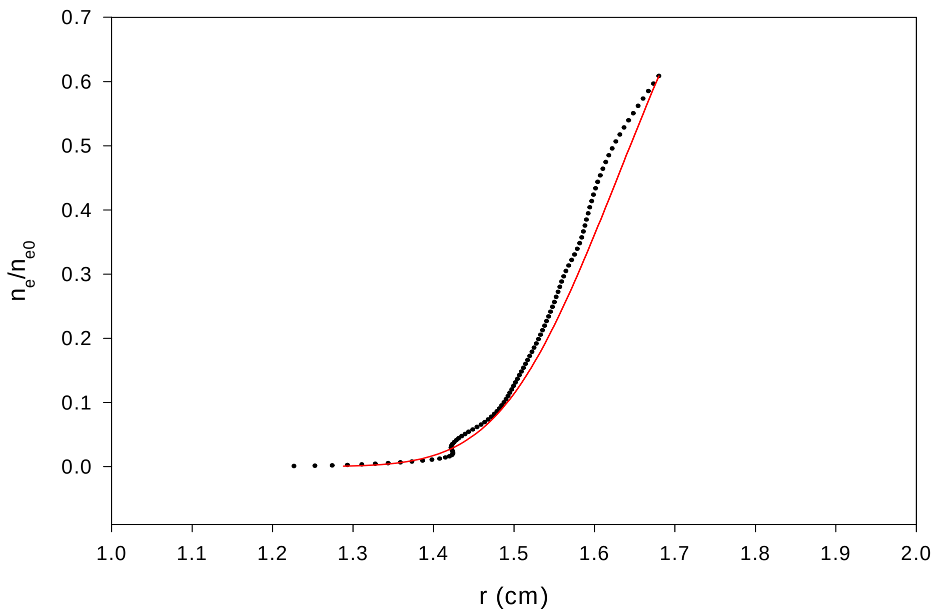


Figure 4

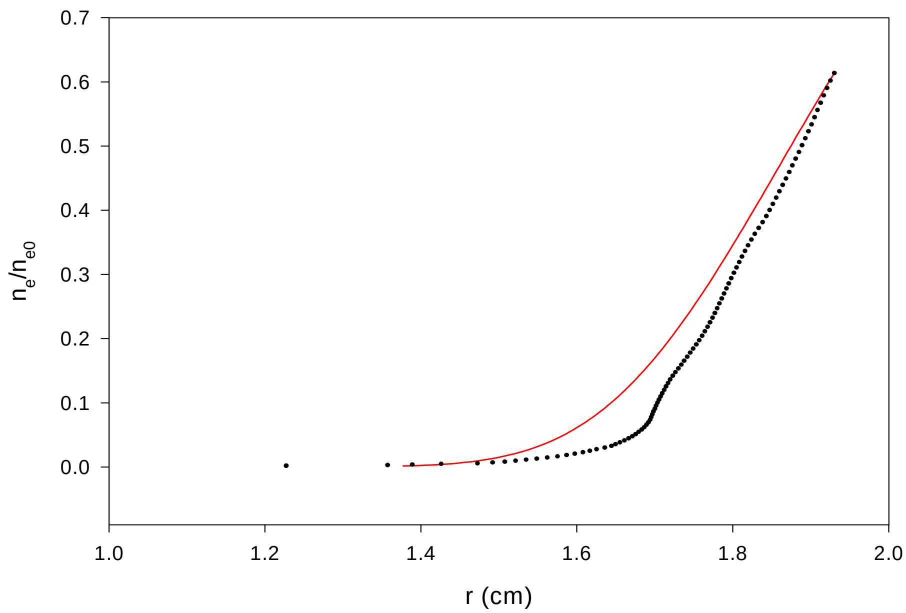


Figure 5

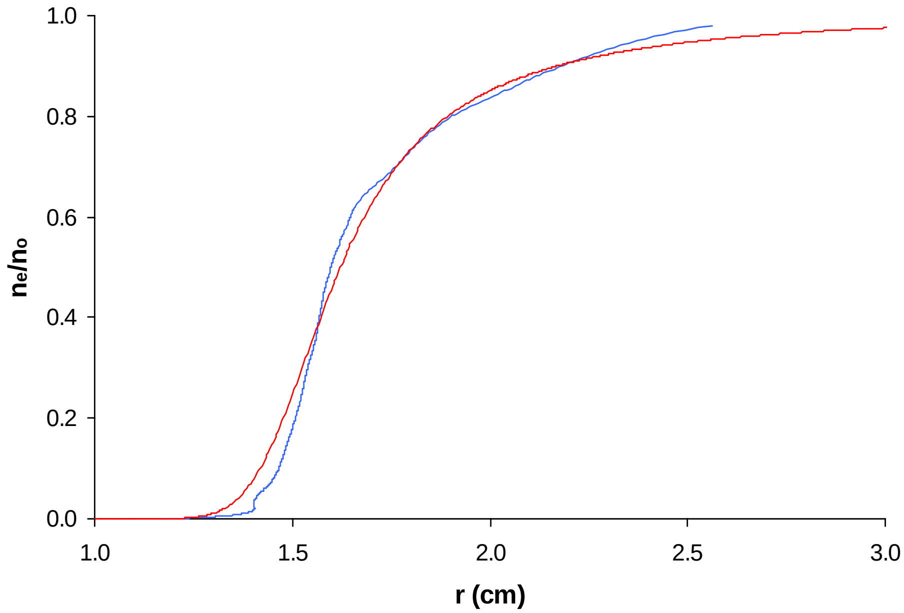


Figure 6